Jungyeon Lee

Jan 5, 2025

Model Predictive Control

Today's Agenda

Machine Learning Control: Overview

Sparse Identification of Nonlinear Dynamics for Model Predictive Controll

Model Predictive Control

Meric Webinar

Model predictive control 는

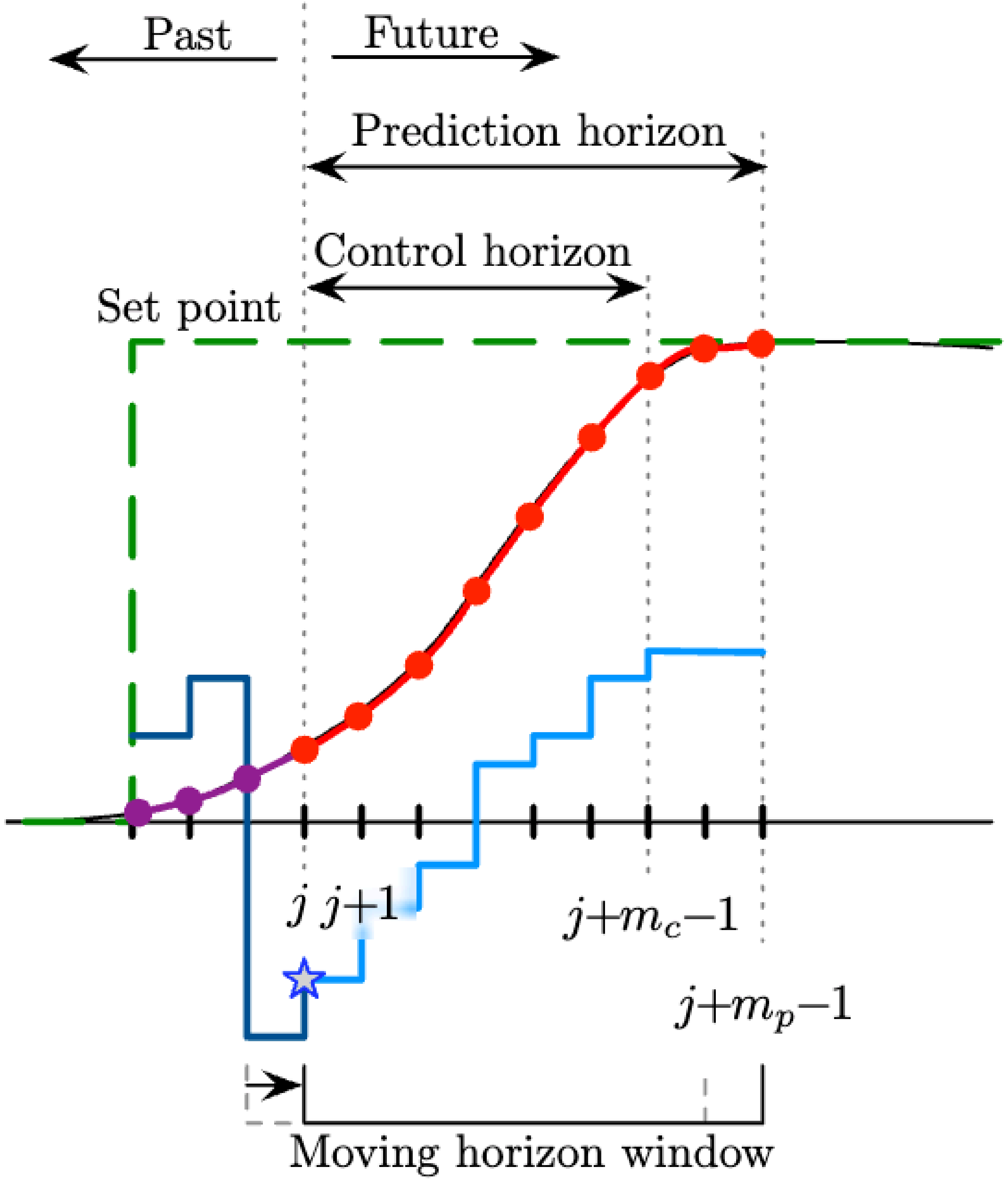
Re

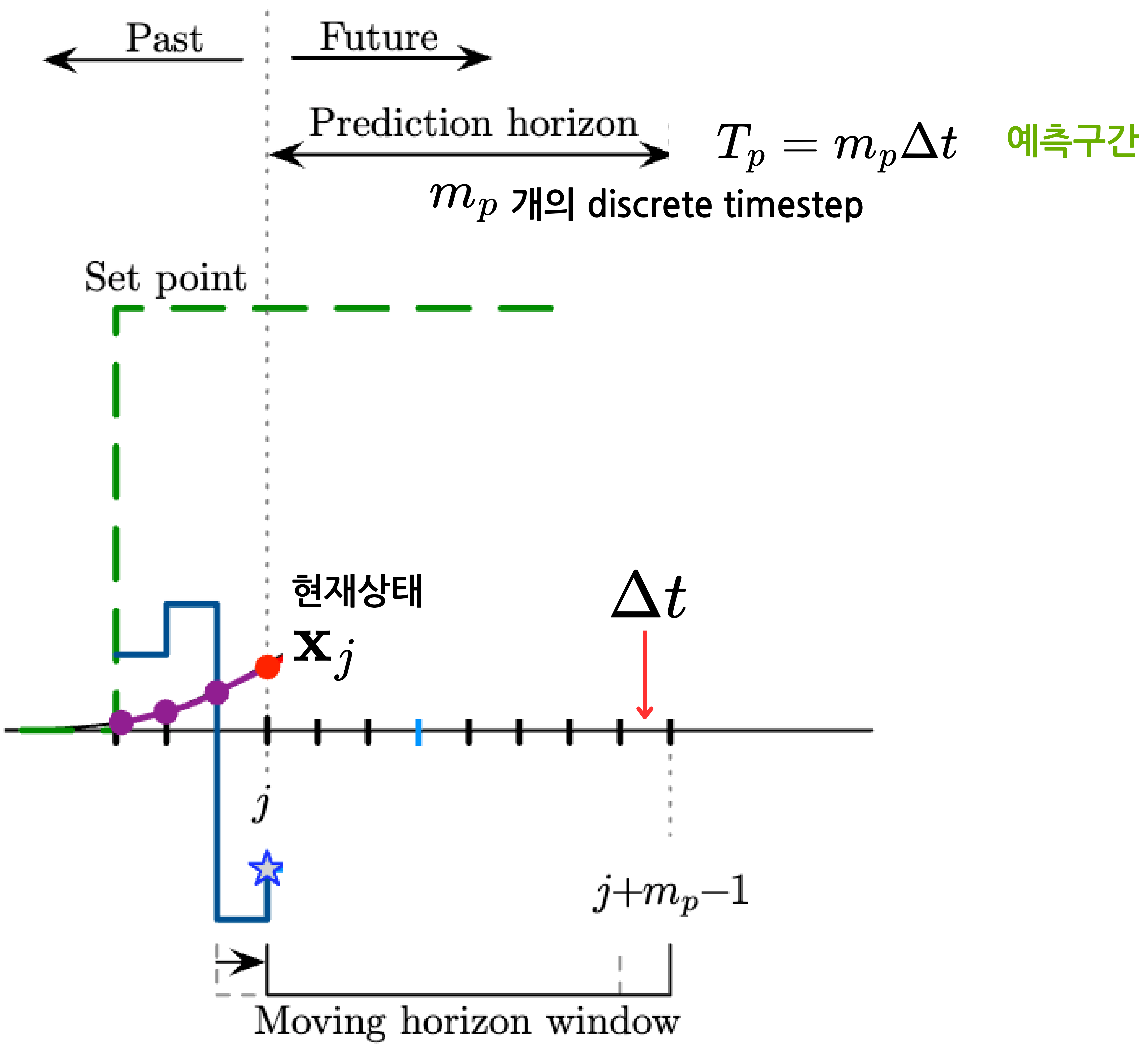
Optimization

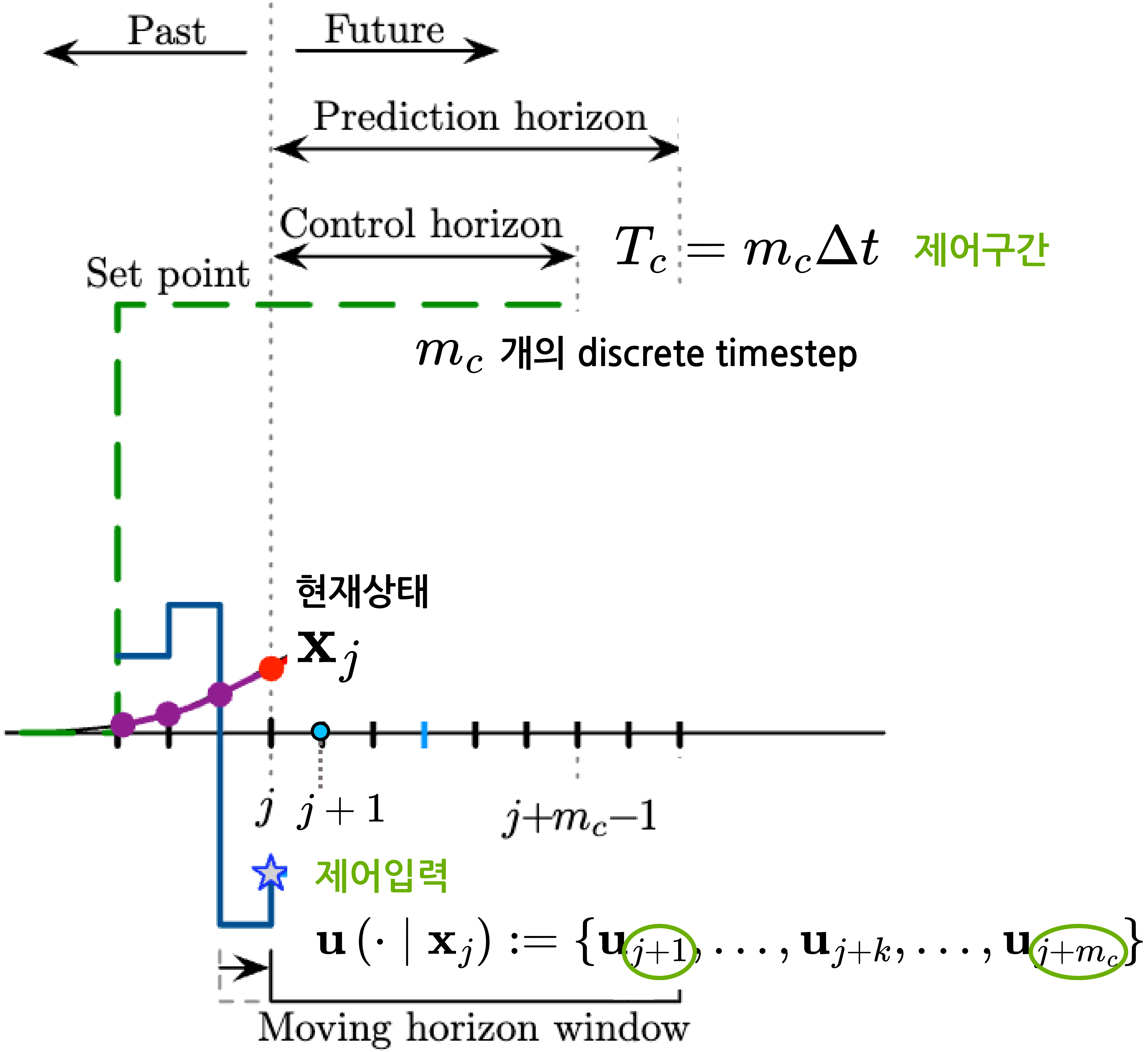
Model predictive control

Model predictive control solves an optimal control problem

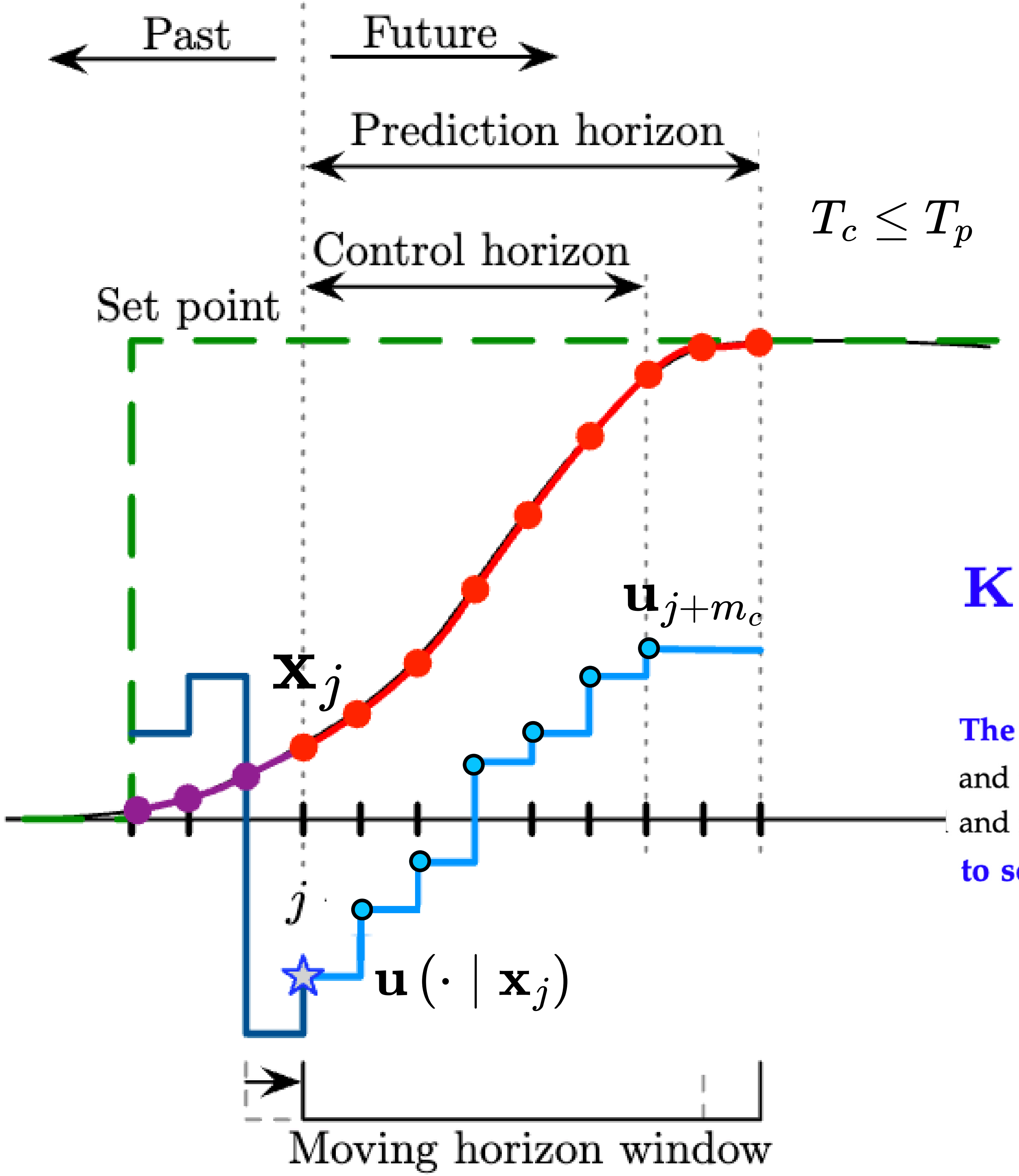
- over a receding horizon, subject to system constraints, to determine the next control action.
- repeated at each new timestep, and the control law is updated
- formulated as an open-loop optimization at each step, which determines the optimal sequence of control inputs over the control horizon




Past
Future
Prediction horizon
$T_p = m_p \Delta t$
예측구간
m_p 개의 discrete timestep
Set point
현재상태
$\mathbf{x}_j$
Δt
j
$j+m_p-1$
Moving horizon window

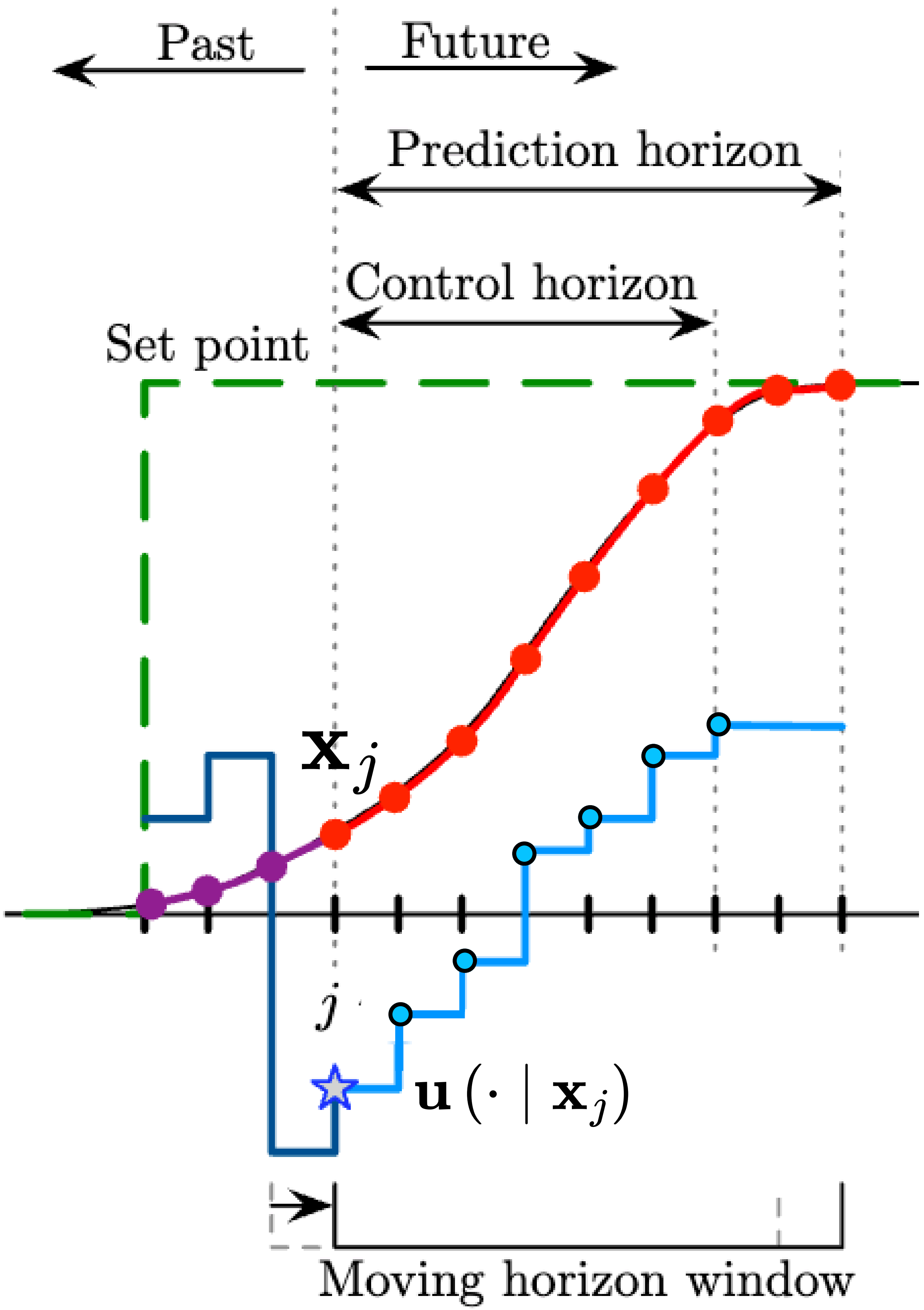


Past
Future
Prediction horizon
Control horizon
$T_c = m_c \Delta t$ 제어구간
Set point
m_c 개의 discrete timestep
현재상태
$\mathbf{x}_j$
j $j+1$ $j+m_c-1$
제어입력
$\mathbf{u}(\cdot \mid \mathbf{x}_j) := \{\mathbf{u}_{j+1}, \ldots, \mathbf{u}_{j+k}, \ldots, \mathbf{u}_{j+m_c}\}$
Moving horizon window



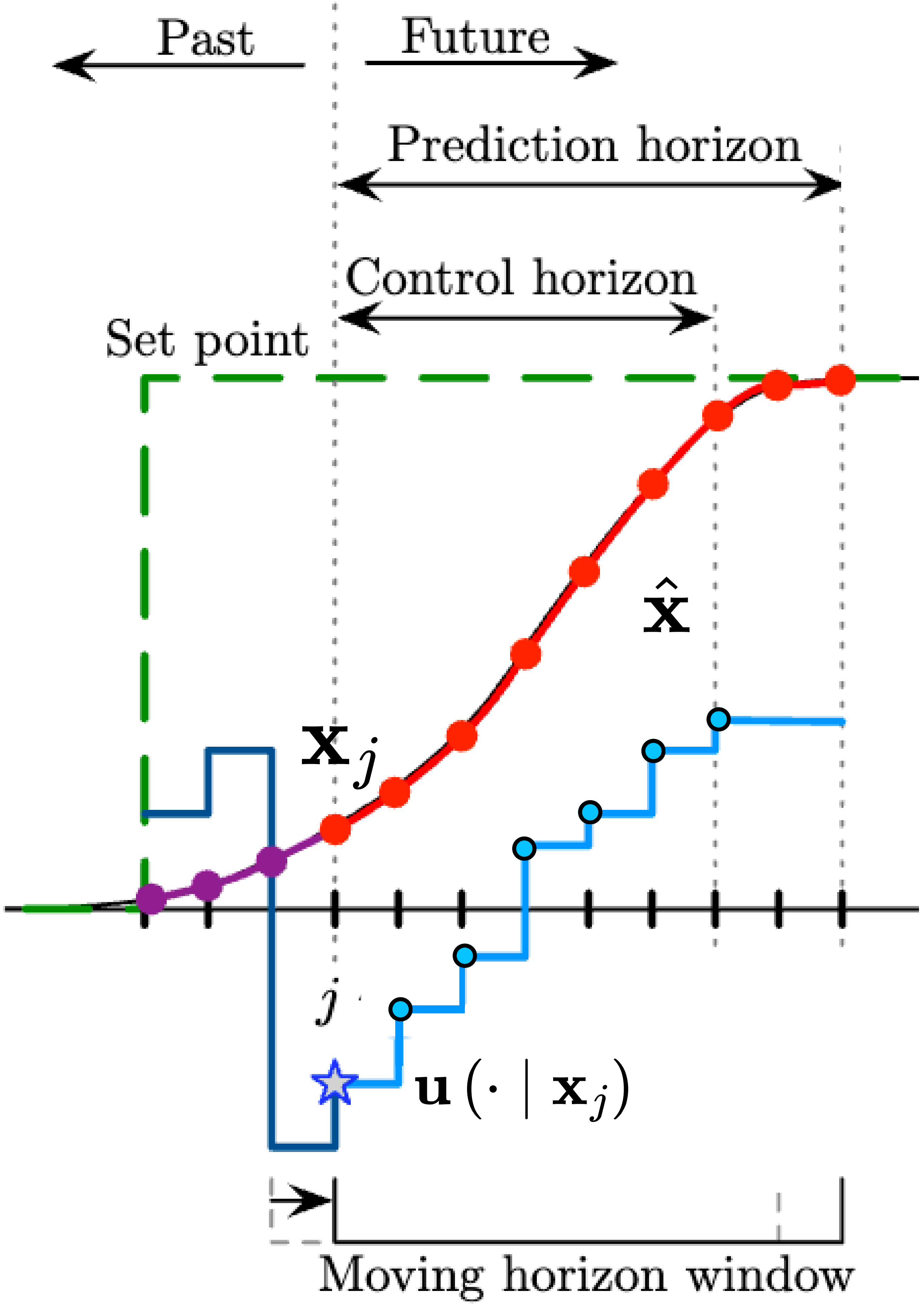
$$\mathbf{K}(\mathbf{x}_j) = \mathbf{u}(j+1|\mathbf{x}_j) = \mathbf{u}_{j+1}$$

The first control value $\mathbf{u}_{j+1}$ is then applied,
and the optimization is reinitialized
and repeated at each subsequent timestep
to solve for the unknown sequence $\mathbf{u}(\cdot|\mathbf{x}_j)$



COST Optimization @ each timestep

$$\min_{\hat{\mathbf{u}}(\cdot|\mathbf{x}_j)} J(\mathbf{x}_j) = \min_{\hat{\mathbf{u}}(\cdot|\mathbf{x}_j)} \Bigg[\left\| \hat{\mathbf{x}}_{j+m_p} - \mathbf{x}^*_{m_p} \right\|^2_{\mathbf{Q}_{m_p}} + \sum_{k=0}^{m_p-1} \left\| \hat{\mathbf{x}}_{j+k} - \mathbf{x}^*_k \right\|^2_{\mathbf{Q}} + \sum_{k=1}^{m_c-1} \left(\left\| \hat{\mathbf{u}}_{j+k} \right\|^2_{\mathbf{R}_u} + \left\| \Delta \hat{\mathbf{u}}_{j+k} \right\|^2_{\mathbf{R}_{\Delta u}} \right) \Bigg]$$



*weight matrices

$\mathbf{Q} \geq 0 \quad \mathbf{Q}_{m_p} \geq 0$

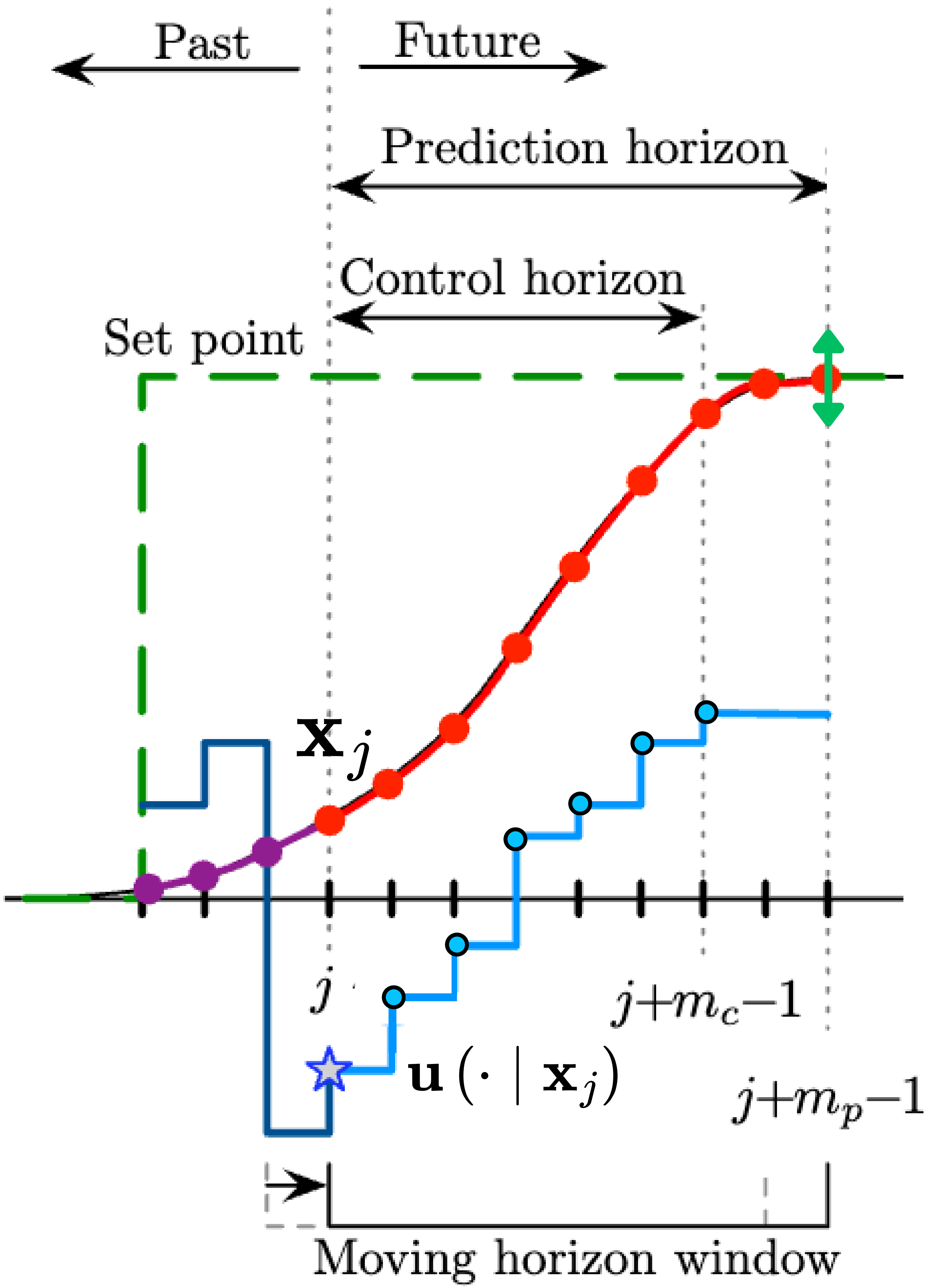
$\mathbf{R}_u > 0 \quad \mathbf{R}_{\Delta u} > 0$

$* \|\mathbf{x}\|_{\mathbf{Q}}^2 := \mathbf{x}^T \mathbf{Q} \mathbf{x}$

$$\min_{\hat{\mathbf{u}}(\cdot|\mathbf{x}_j)} J(\mathbf{x}_j) = \min_{\hat{\mathbf{u}}(\cdot|\mathbf{x}_j)} \left[\left\| \hat{\mathbf{x}}_{j+m_p} - \mathbf{x}_{m_p}^* \right\|_{\mathbf{Q}_{m_p}}^2 + \sum_{k=0}^{m_p-1} \left\| \hat{\mathbf{x}}_{j+k} - \mathbf{x}_k^* \right\|_{\mathbf{Q}}^2 + \sum_{k=1}^{m_c-1} \left(\left\| \hat{\mathbf{u}}_{j+k} \right\|_{\mathbf{R}_u}^2 + \left\| \Delta \hat{\mathbf{u}}_{j+k} \right\|_{\mathbf{R}_{\Delta u}}^2 \right) \right]$$

discrete-time system dynamics $\hat{\mathbf{F}} : \mathbb{R}^n \times \mathbb{R}^q \to \mathbb{R}^n$

$\longrightarrow \hat{\mathbf{x}}_{k+1} = \hat{\mathbf{F}}(\hat{\mathbf{x}}_k, \mathbf{u}_k)$



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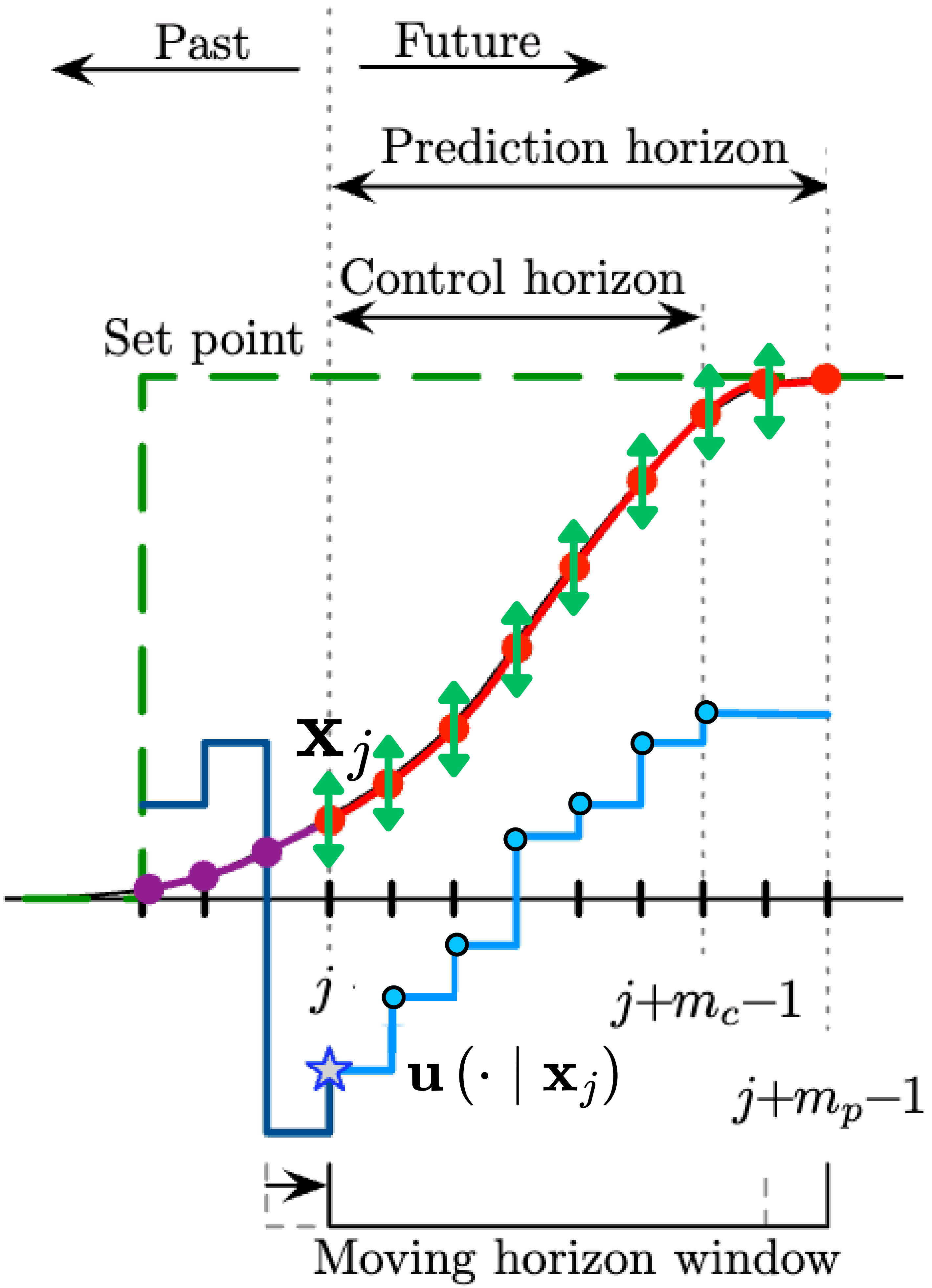
Prediction 끝

$$\min_{\hat{\mathbf{u}}(\cdot|\mathbf{x}_j)} J(\mathbf{x}_j) = \min_{\hat{\mathbf{u}}(\cdot|\mathbf{x}_j)} \Bigg[\left\| \hat{\mathbf{x}}_{j+m_p} - \mathbf{x}^*_{m_p} \right\|^2_{\mathbf{Q}_{m_p}} + \sum_{k=0}^{m_p-1} \left\| \hat{\mathbf{x}}_{j+k} - \mathbf{x}^*_k \right\|^2_{\mathbf{Q}} + \sum_{k=1}^{m_c-1} \left(\left\| \hat{\mathbf{u}}_{j+k} \right\|^2_{\mathbf{R}_u} + \left\| \Delta \hat{\mathbf{u}}_{j+k} \right\|^2_{\mathbf{R}_{\Delta u}} \right) \Bigg]$$



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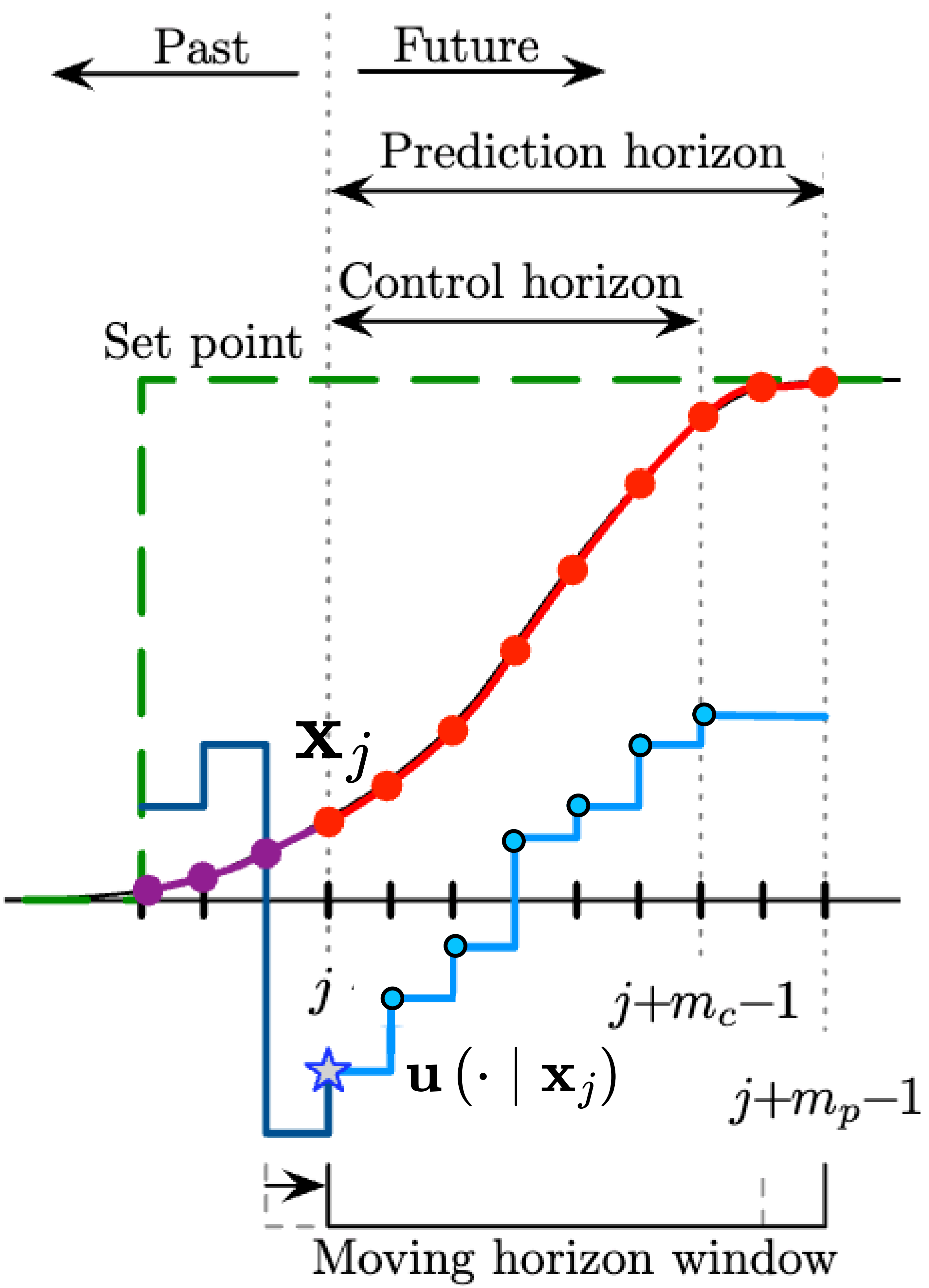
Prediction 끝 제외
모든 prediction timestep



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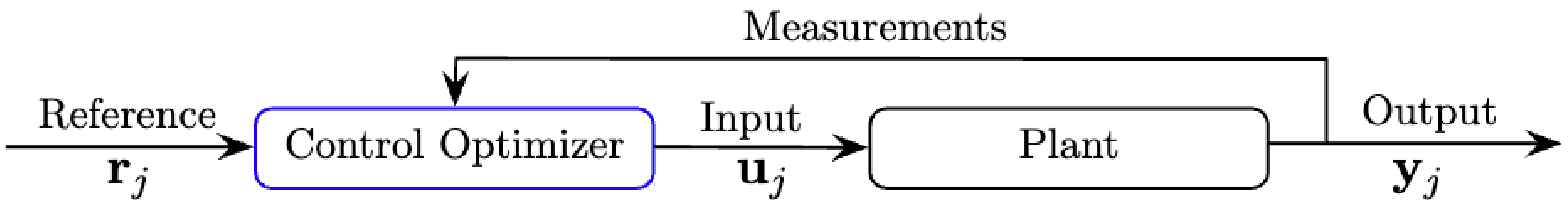
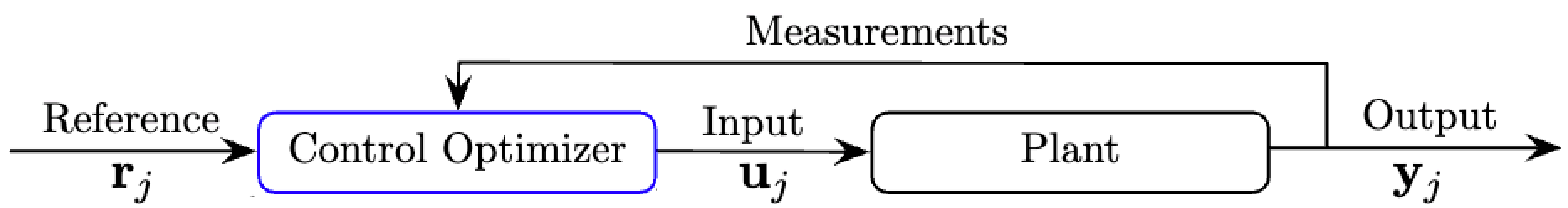
Control timestep에서(j+1부터 시작)

def. $\Delta \mathbf{u}_k = \mathbf{u}_k - \mathbf{u}_{k-1}$

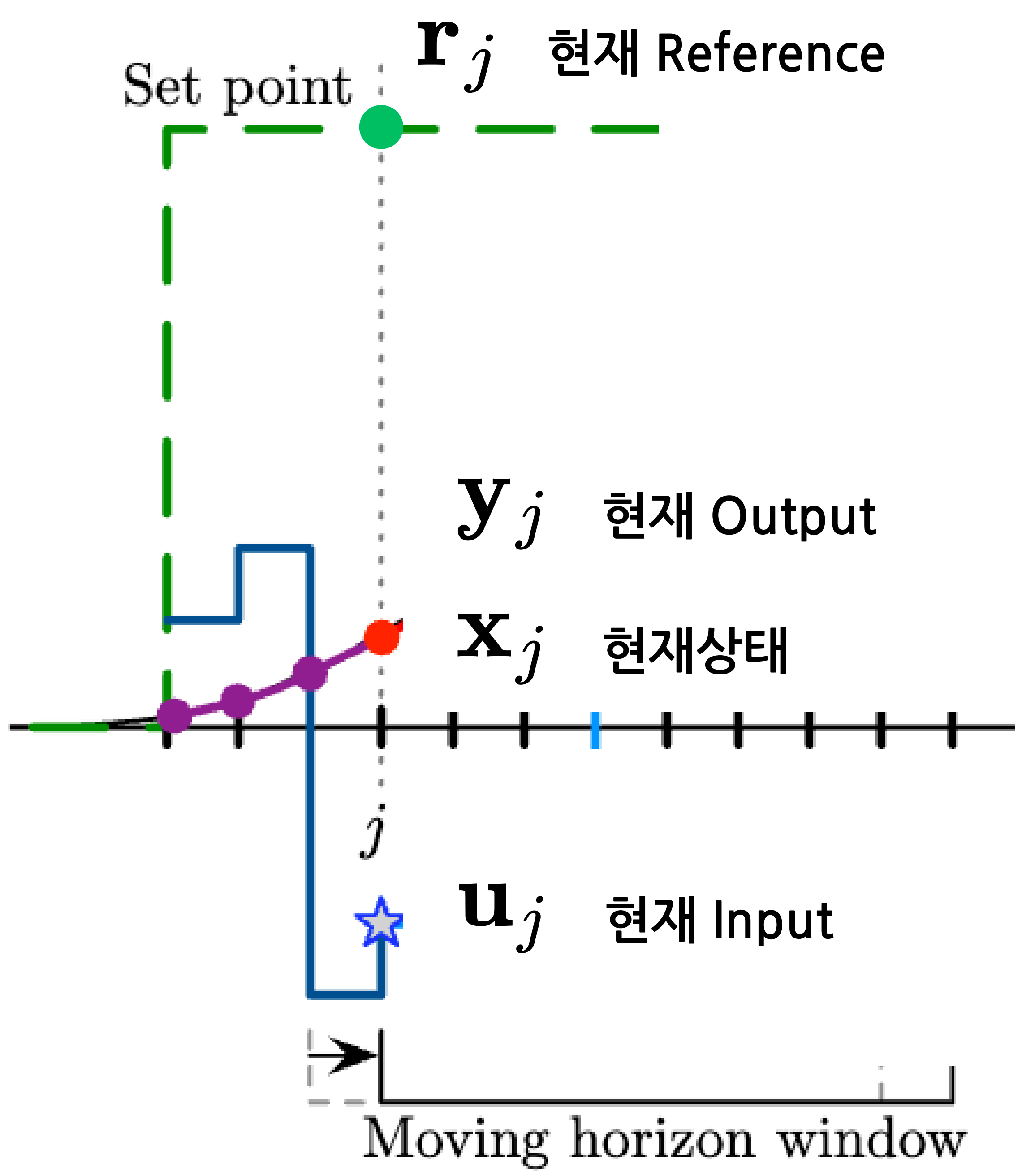
input constraints $\Delta \mathbf{u}_{\min} \leq \Delta \mathbf{u}_k \leq \Delta \mathbf{u}_{\max}$

$\mathbf{u}_{\min} \leq \mathbf{u}_k \leq \mathbf{u}_{\max}$

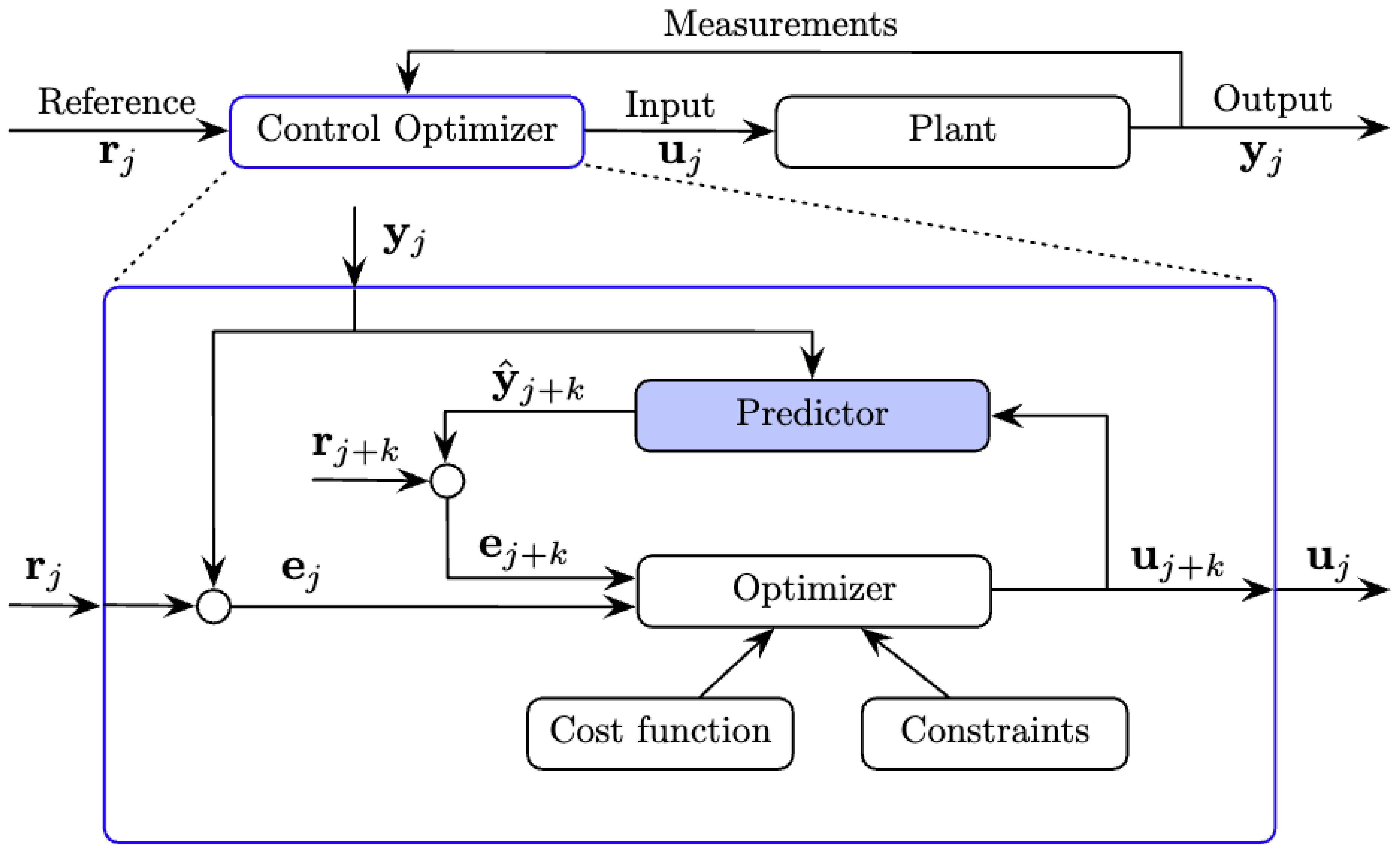

Measurements
Reference
$\mathbf{r}_j$
Control Optimizer
Input
$\mathbf{u}_j$
Plant
Output
$\mathbf{y}_j$

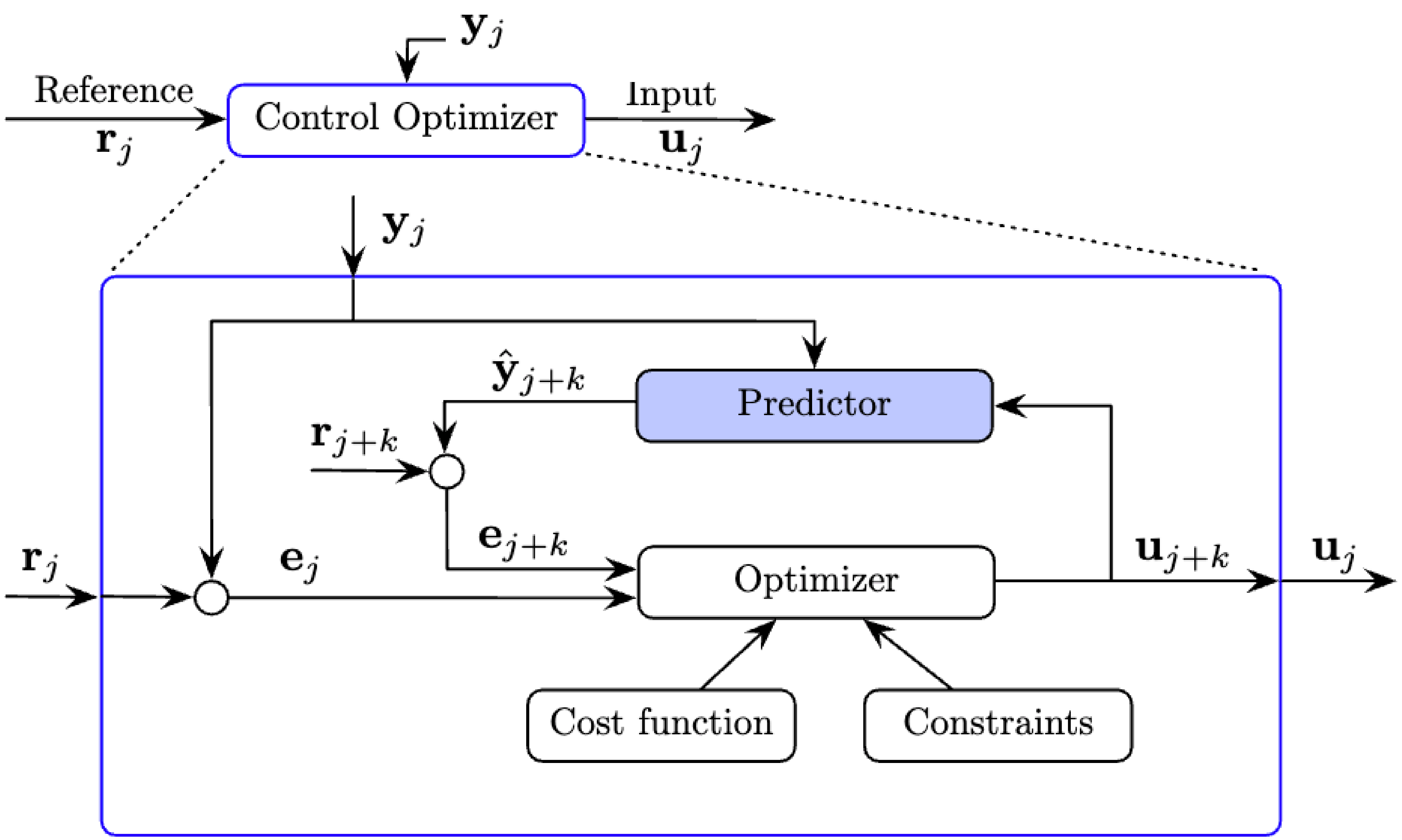


Measurements
Reference
$\mathbf{r}_j$
Control Optimizer
Input
$\mathbf{u}_j$
Plant
Output
$\mathbf{y}_j$

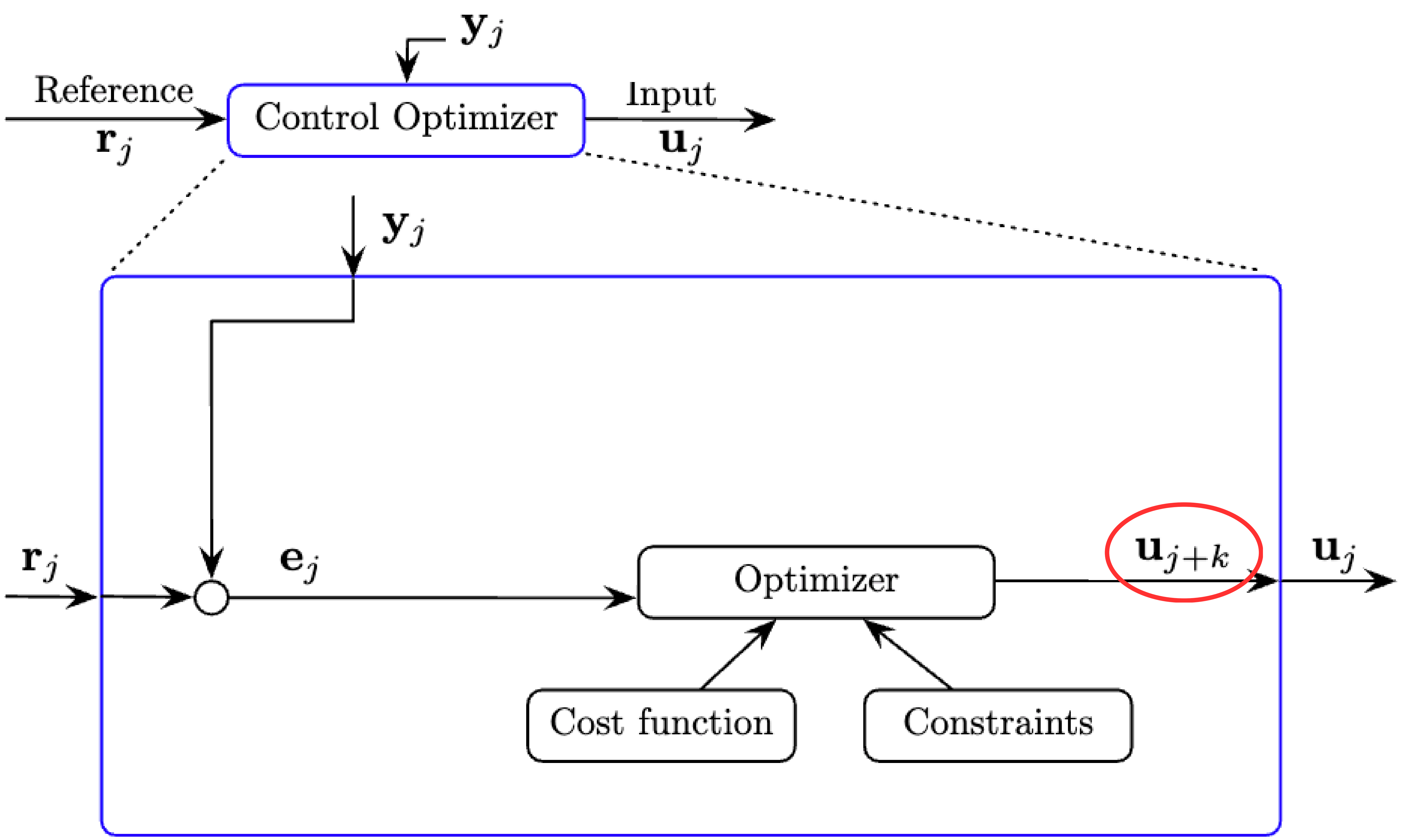


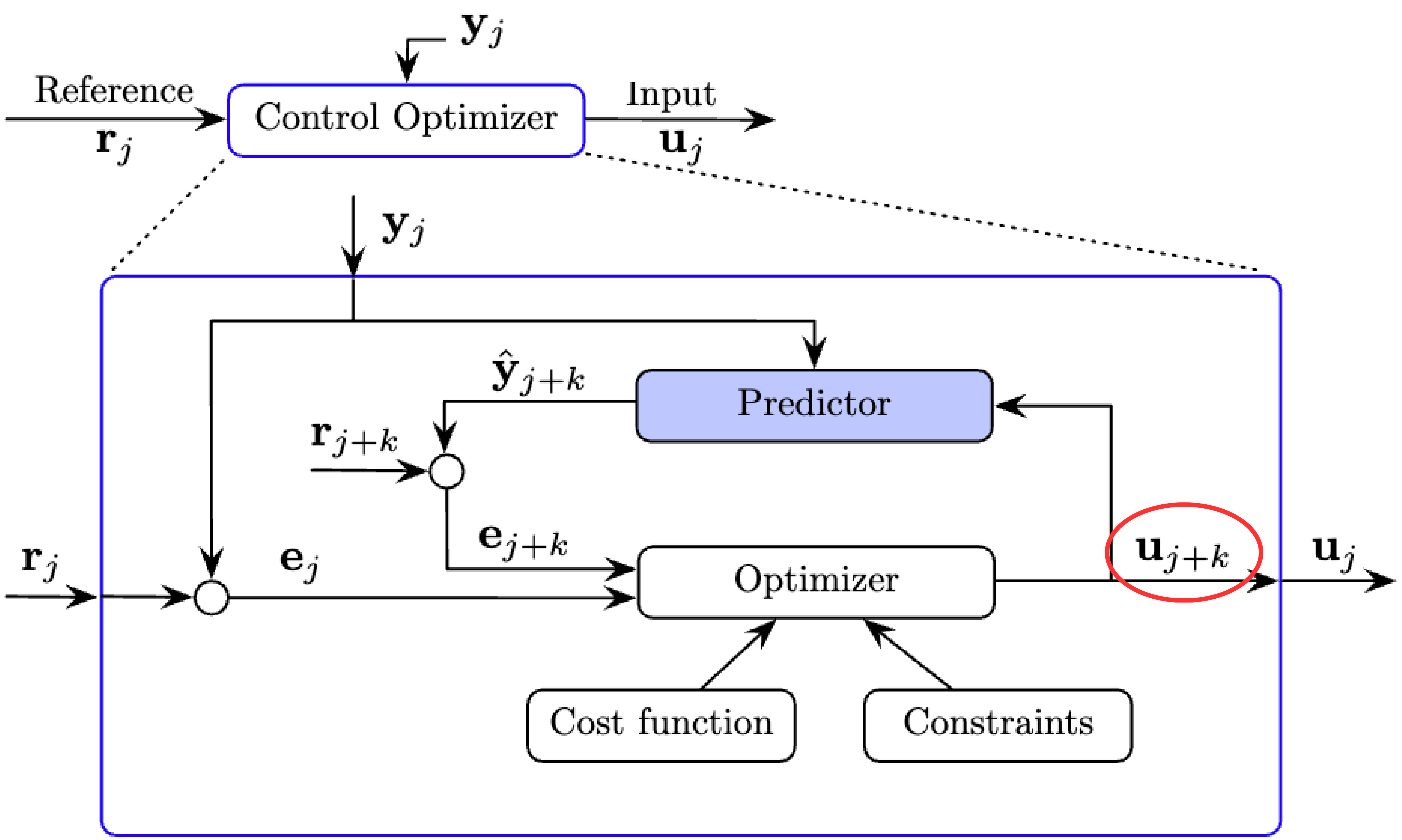
$\mathbf{r}_j$ 현재 Reference
Set point
$\mathbf{y}_j$ 현재 Output
$\mathbf{x}_j$ 현재상태
j
$\mathbf{u}_j$ 현재 Input
Moving horizon window


Measurements
Reference
$\mathbf{r}_j$
Control Optimizer
Input
$\mathbf{u}_j$
Plant
Output
$\mathbf{y}_j$
$\mathbf{y}_j$
$\hat{\mathbf{y}}_{j+k}$
Predictor
$\mathbf{r}_{j+k}$
$\mathbf{e}_{j+k}$
$\mathbf{r}_j$
$\mathbf{e}_j$
Optimizer
$\mathbf{u}_{j+k}$
$\mathbf{u}_j$
Cost function
Constraints


$\mathbf{y}_j$
Reference
$\mathbf{r}_j$
Control Optimizer
Input
$\mathbf{u}_j$
$\mathbf{y}_j$
$\hat{\mathbf{y}}_{j+k}$
Predictor
$\mathbf{r}_{j+k}$
$\mathbf{e}_{j+k}$
$\mathbf{r}_j$
$\mathbf{e}_j$
Optimizer
$\mathbf{u}_{j+k}$
$\mathbf{u}_j$
Cost function
Constraints

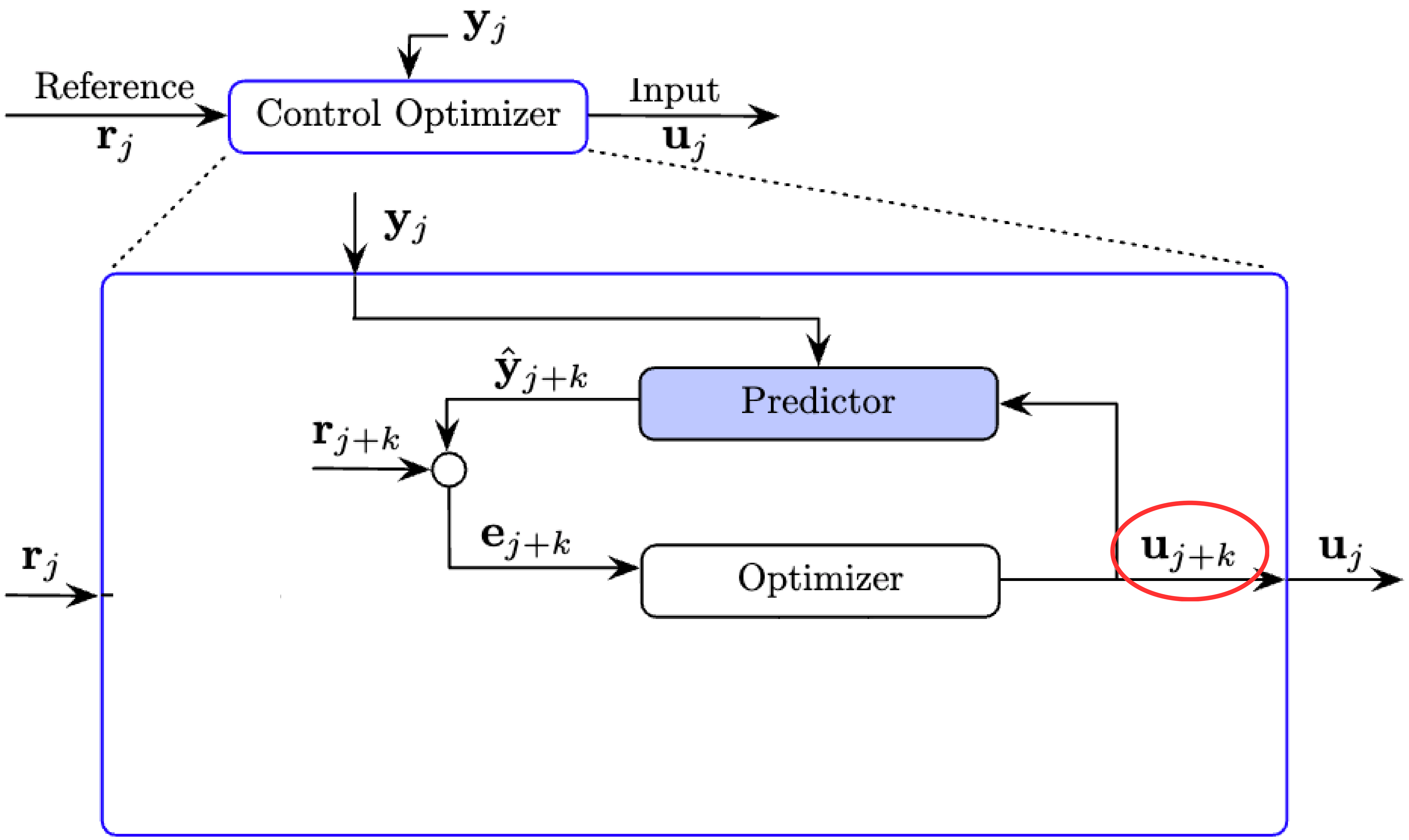
Optimizer 관점에서



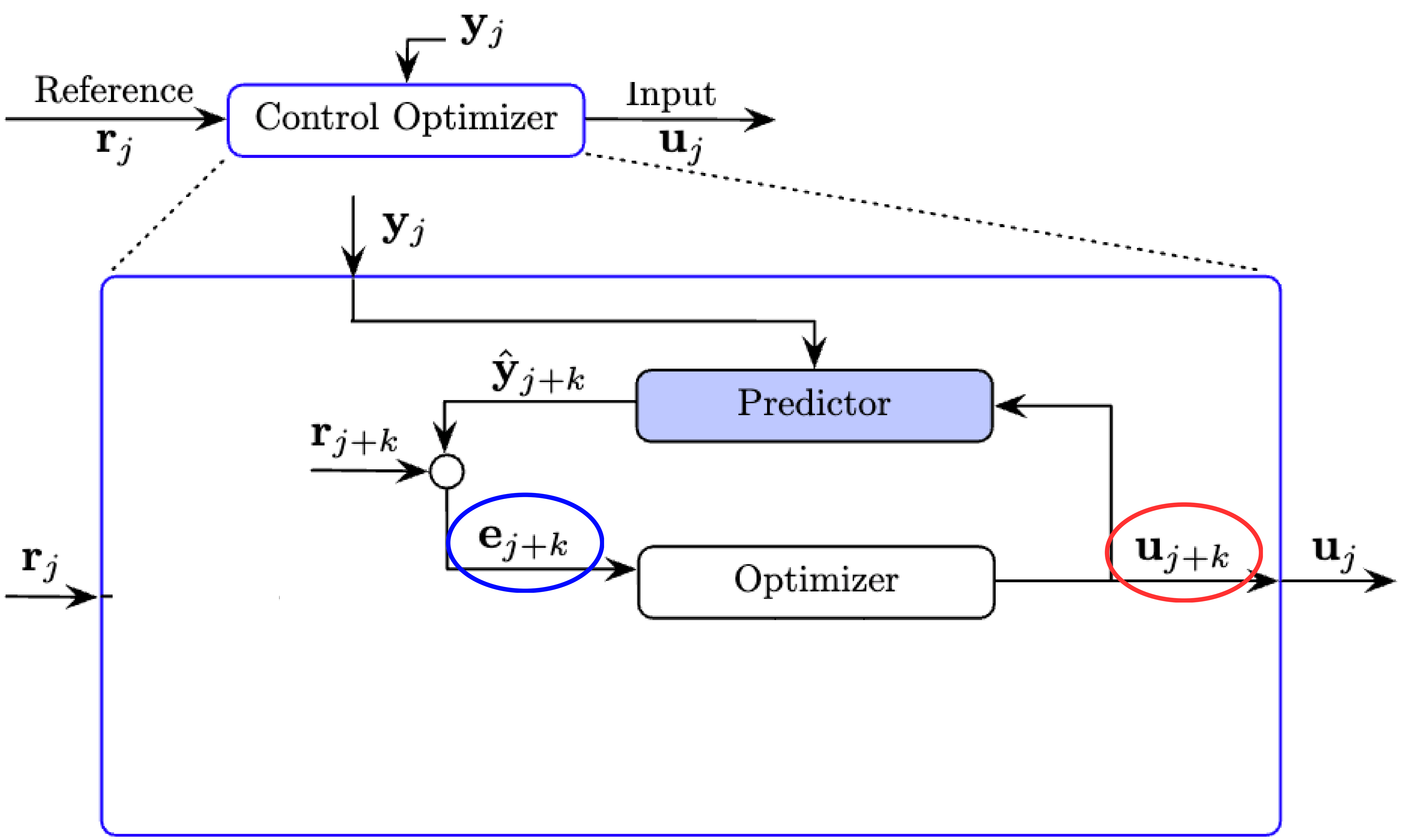


$\mathbf{y}_j$
Reference
$\mathbf{r}_j$
Control Optimizer
Input
$\mathbf{u}_j$
$\mathbf{y}_j$
$\hat{\mathbf{y}}_{j+k}$
Predictor
$\mathbf{r}_{j+k}$
$\mathbf{e}_{j+k}$
$\mathbf{r}_j$
$\mathbf{e}_j$
Optimizer
$\mathbf{u}_{j+k}$
$\mathbf{u}_j$
Cost function
Constraints

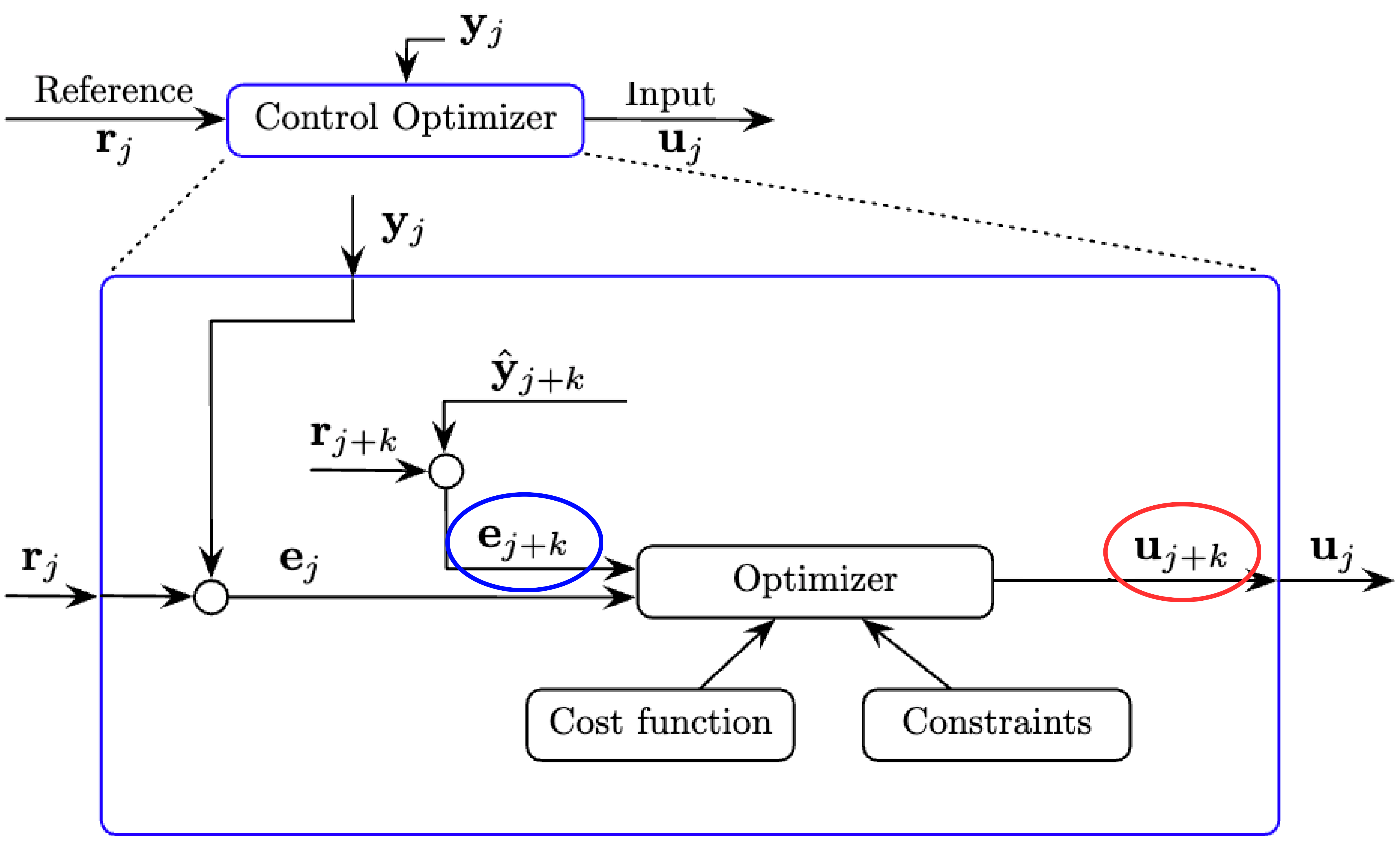
Predictor 관점에서



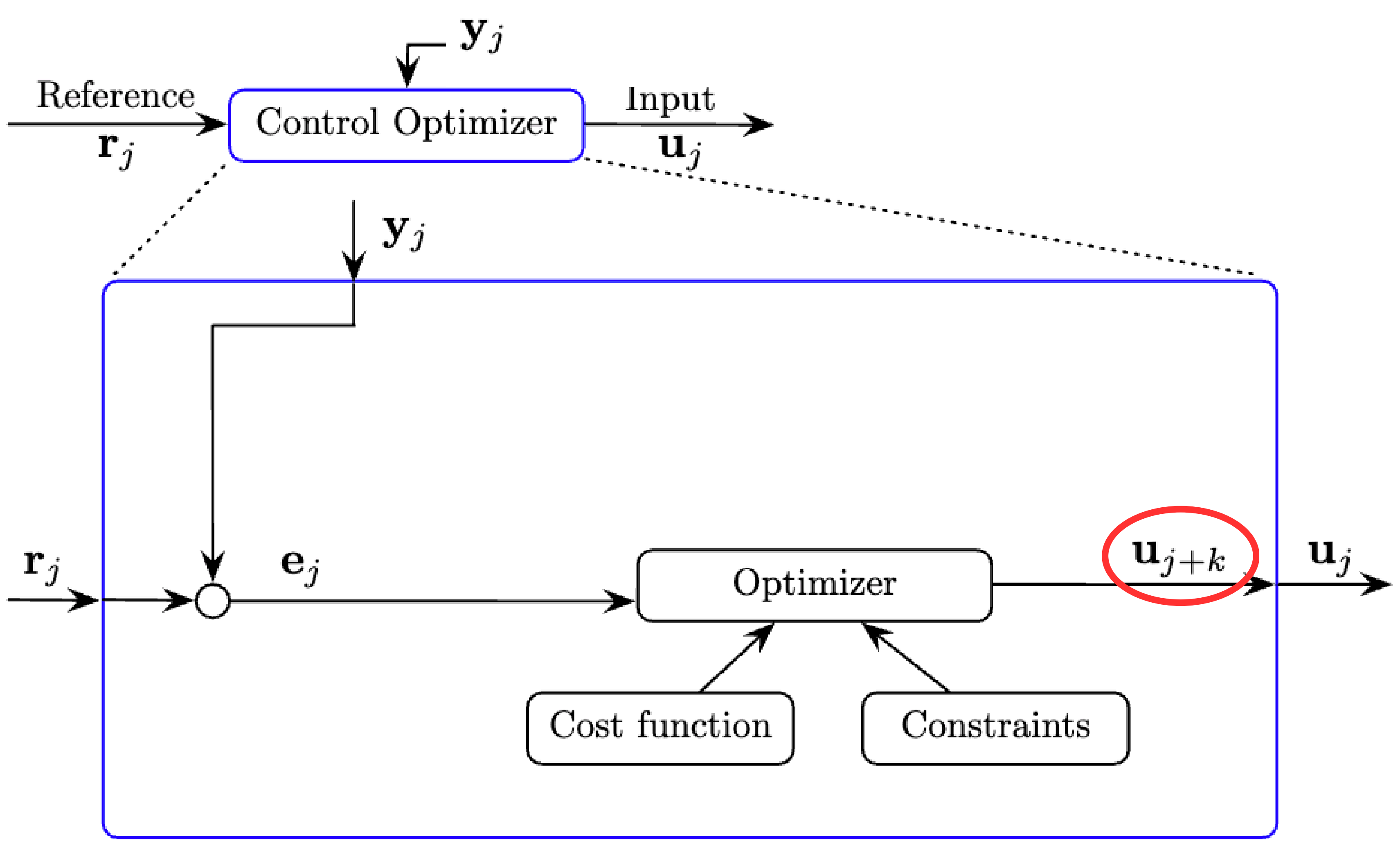
Predictor 관점에서



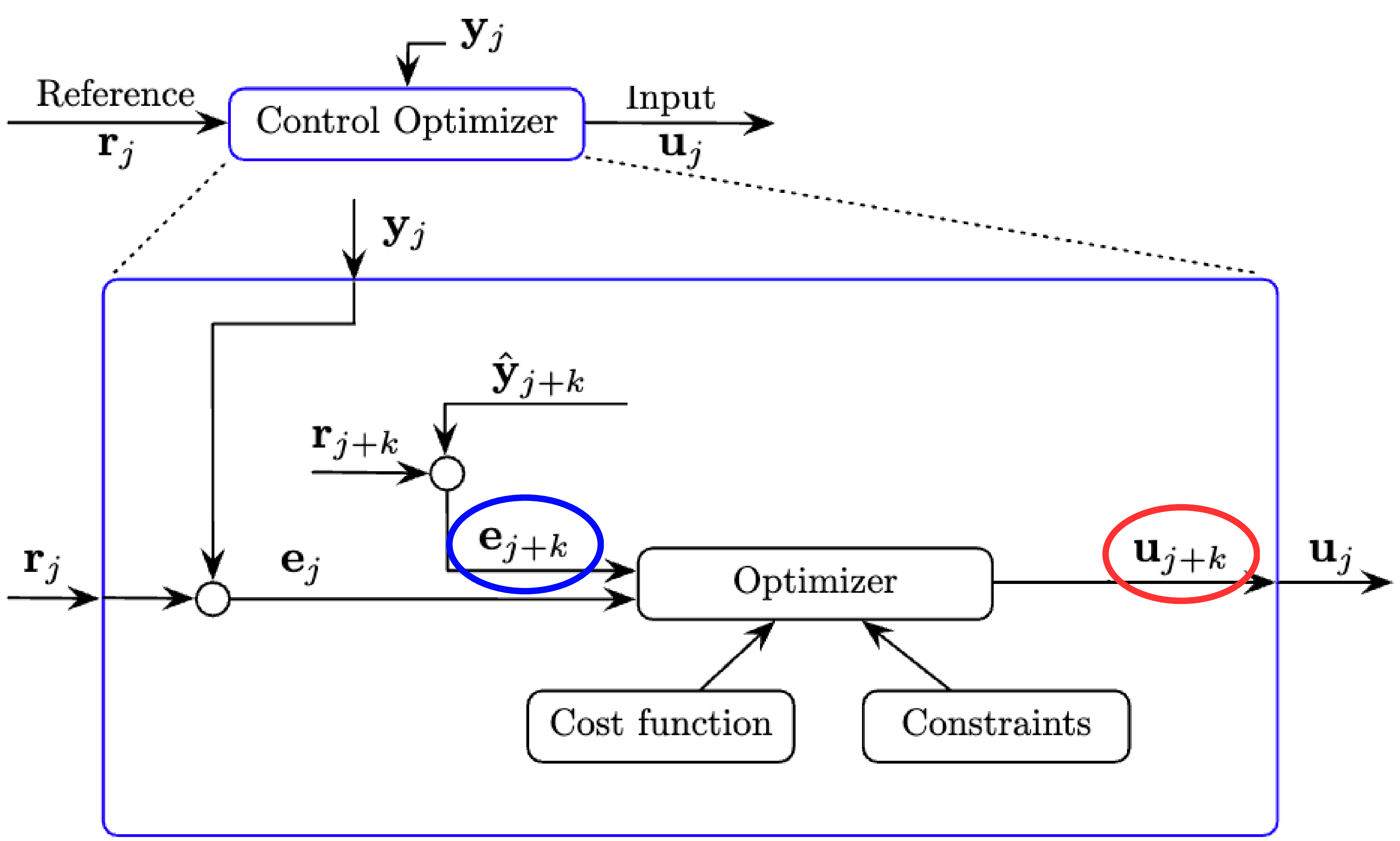
다시 Optimizer 관점에서



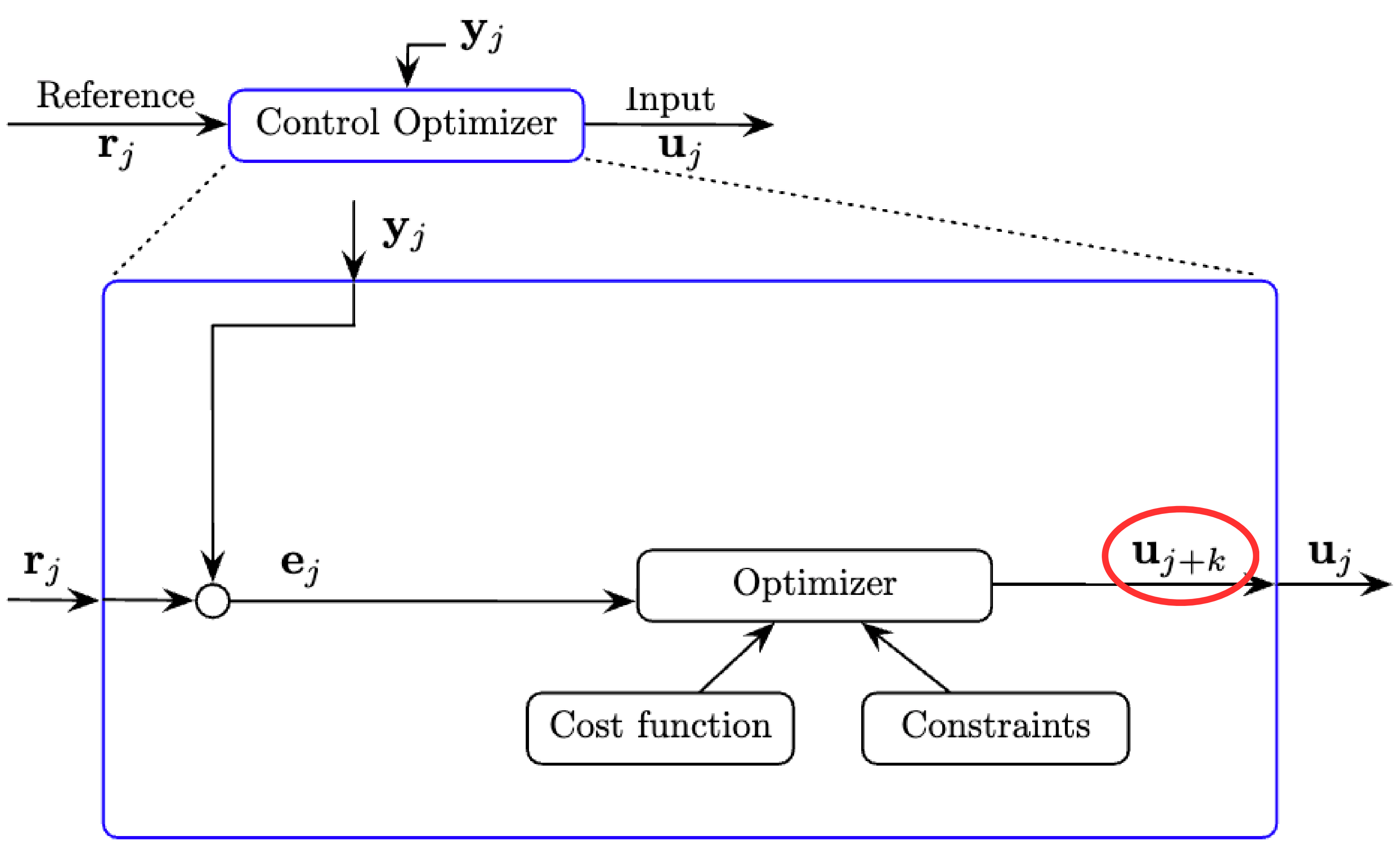
Predictor X



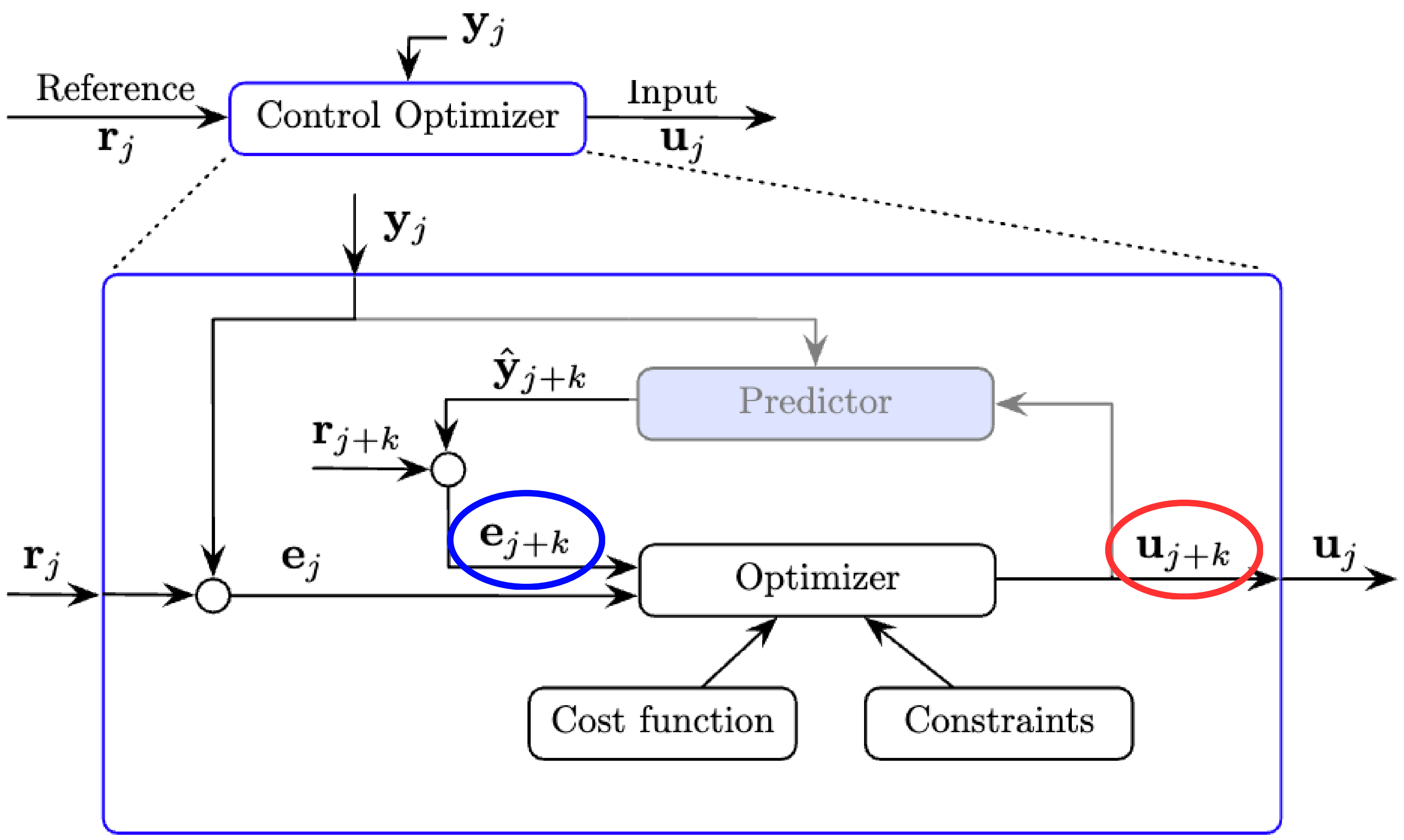
Predictor O



Predictor X



Predictor O



QP solution

- QP Problem:

$$AU \le b$$

$$J = \frac{1}{2}U^T Q U + f^T U \to \min$$

$U = U(t)$ Predicted control sequence

$$Q = rD^T D + H^T H$$

$$f = H^T (Gx + Fu)$$

$$A = \begin{bmatrix} I \\ -I \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \cdot u_0$$

```python
    # dynamic constraints: \dot{x} = A_{c}x + B_{c}u
    def _generate_state_space_model(self):
        # Ac (13 * 13), Bc (13 * 12)
        Ac = np.zeros((self.num_state, self.num_state), dtype=np.float32)
        Bc = np.zeros((self.num_state, self.num_input), dtype=np.float32)

        Rz = np.array([[np.cos(self.yaw), -np.sin(self.yaw), 0],
                       [np.sin(self.yaw), np.cos(self.yaw), 0],
                       [0, 0, 1]], dtype=np.float32)
        # Rz = self.__robot_data.R_base
        world_I = Rz @ self.base_inertia_base @ Rz.T

        Ac[0:3, 6:9] = Rz.T
        Ac[3:6, 9:12] = np.identity(3, dtype=np.float32)
        Ac[11, 12] = 1.0

        for i in range(4):
            Bc[6:9, 3*i:3*i+3] = np.linalg.inv(world_I) @ vec2so3(self.pos_base_feet[i])
            Bc[9:12, 3*i:3*i+3] = np.identity(3, dtype=np.float32) / self.mass

        return Ac, Bc
```